Optimizing the Error Recovery Capabilities of LDPC-staircase Codes Featuring a Gaussian Elimination Decoding

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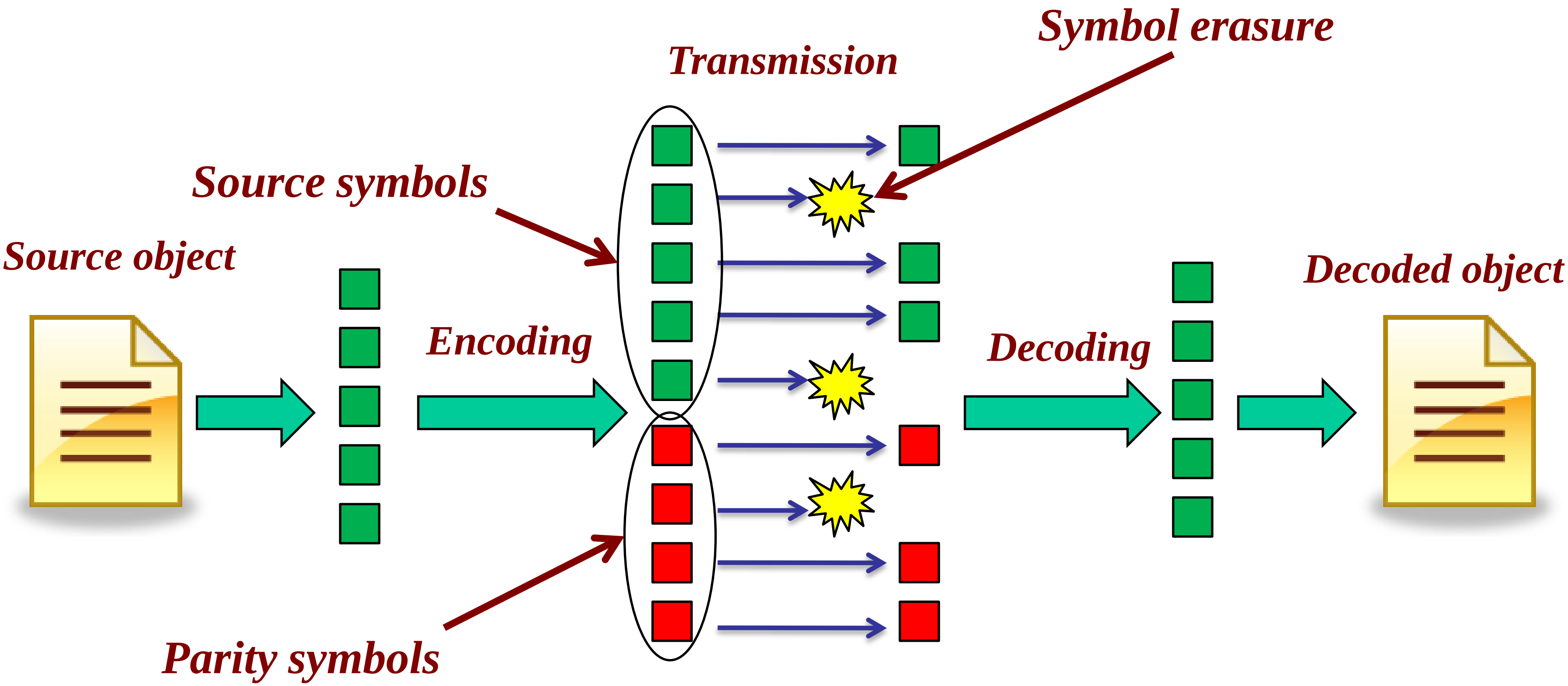
Introduction

- LDPC-staircase/triangle codes
  - Forward Error Correction (FEC) codes for the erasure channel
  - Extremely efficient
  - Now an IETF standard (RFC5170) http://www.rfc-editor.org/rfc/rfc5170.txt
  - Open source codec available http://planete-bcast.inrialpes.fr

What is a FEC code for the erasure channel?

- Source object divided into k symbols 🟩 🟩 🟩 🟩 🟩
- Encoding: add redundancy = (n-k) parity symbols 🟥 🟥 🟥 🟥
- Decoding: rebuild the source object from the k(1+ε) symbols received



Some more details on LDPC codes considered

- Parity check matrix of LDPC-Staircase
  - Relation between source and parity symbols

Source symbols **Parity symbols**

$$\begin{array}{ccccc|ccccc}
S_1 & S_2 & S_3 & S_4 & S_5 & P_1 & P_2 & P_3 & P_4 & P_5 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}$$

Constraints

$$S_1 \oplus S_4 \oplus S_5 \oplus P_1 \oplus P_2 = 0$$

- Encoding
  - Create parity symbols

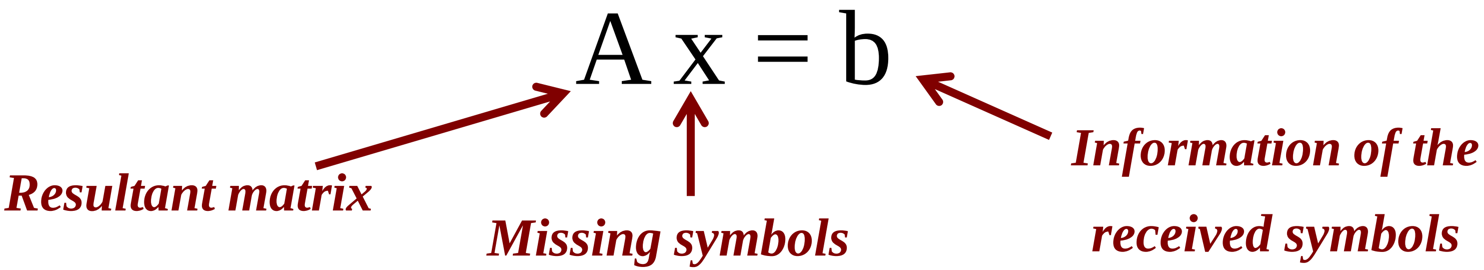
$$S_3 \oplus S_4 \oplus P_1 = 0\,P_1$$
$$S_1 \oplus S_4 \oplus S_5 \oplus P_1 \oplus P_2 = 0\,P_2$$
$$S_1 \oplus S_2 \oplus S_3 \oplus P_2 \oplus P_3 = 0\,P_3$$
$$S_2 \oplus S_4 \oplus S_5 \oplus P_3 \oplus P_4 = 0\,P_4$$
$$S_1 \oplus S_2 \oplus S_3 \oplus S_5 \oplus P_4 \oplus P_5 = 0\,P_5$$

Some more details on LDPC codes considered

- Decoding
  - solve a system of linear equations
  - Several techniques are feasible…
- Sol.1: Iterative Decoding (ID)
  - If an equation has only one unknown variable, this latter is equal to the sum of the others. Reiterate …
  - Pros: Low complexity (linear)
    - **Low CPU load and high sustainable bandwidth**
  - Cons: Suboptimal in terms of correction capabilities
    - **Some systems cannot be solved**

Some more details on LDPC codes considered

- Sol.2: Gaussian Elimination (GE) decoding
  - Solve a linear system



$$A\,x = b$$

Resultant matrix

Missing symbols

Information of the received symbols

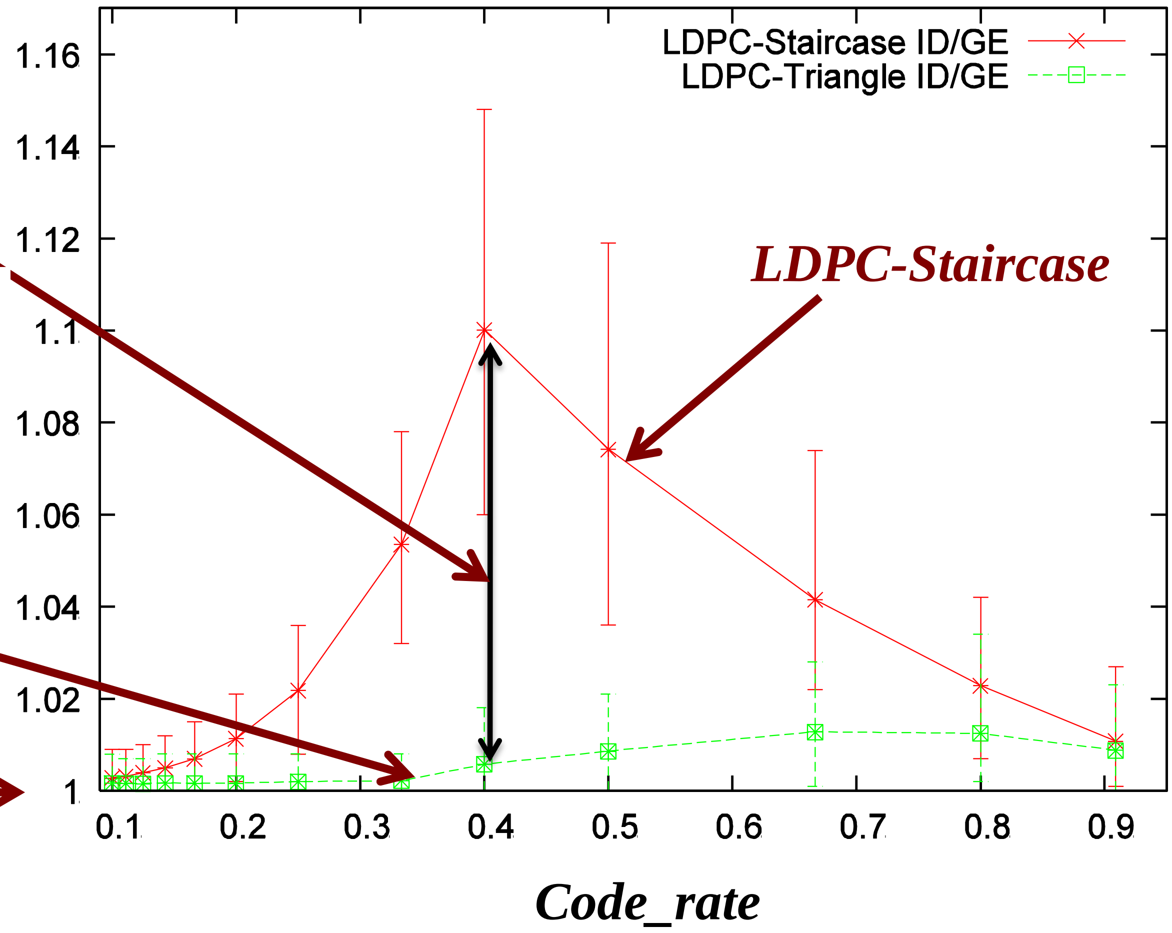
  - Optimal erasure correction capabilities
    - **Maximum Likelihood (ML) decoding**
  - Often believed as too costly to be used…
    - **But is it really the case ? ☺**

Some more details on LDPC codes considered

- Sol. 3: **Hybrid ID/GE** scheme
  - **start decoding with ID**
  - **finish with GE if necessary**
- Sol. 4: Patented techniques
  - **[Burshtein & Miller, 04]**
  - **Digital fountain: U.S. Patent Number 6,856,263**
- Thanks to Hybrid decoding:
  - **excellent erasure correction capabilities…**
  - **… while remaining very fast**
  - **we'll always consider hybrid decoding in the remaining of the slides!**

LDPC-Triangle vs Staircase erasure recovery

- ○ LDPC-**Triangle** are very close to ideal codes…
- ○ …but there is place for improvement with LDPC-**Staircase**



The gap we need to reduce

Average inefficiency ratio

(the lower, the better)

LDPC-Triangle are

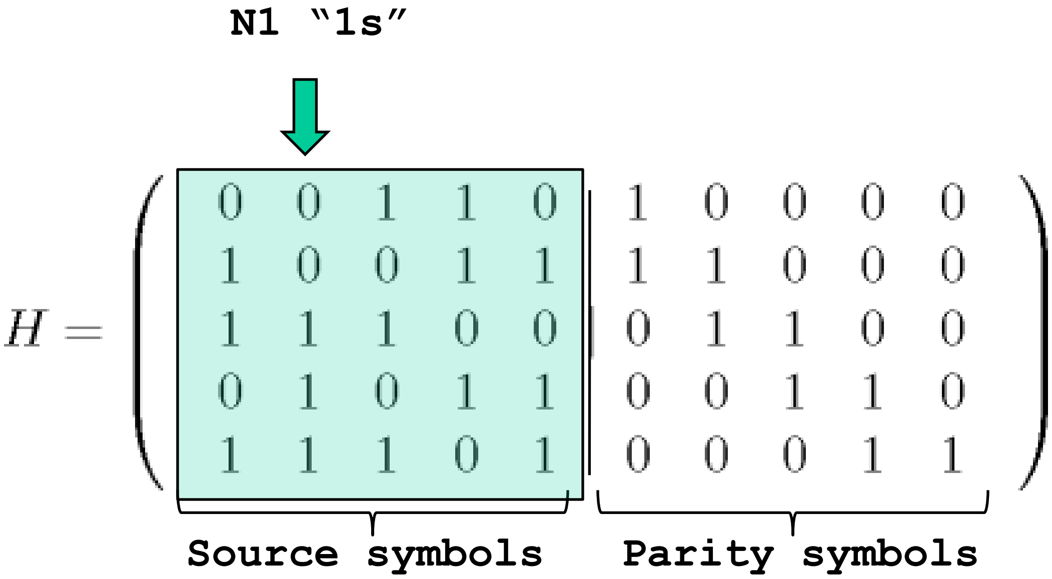
close to ideal codes

Ideal code

Ineff. ratio =1.0

Improving LDPC-Staircase codes

- By adjusting the “N1” parameter
  - **number of “1s” in each column of the left side of the parity check matrix**



N1 “1s”

$$H = \left(\underbrace{\boxed{\begin{matrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{matrix}}}_{\text{Source symbols}} \middle| \underbrace{\begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}}_{\text{Parity symbols}} \right)$$

- **N1 was fixed and equal to 3 until recently**
  - Was meaningful with ID, but not with GE

Improved erasure correction capabilities

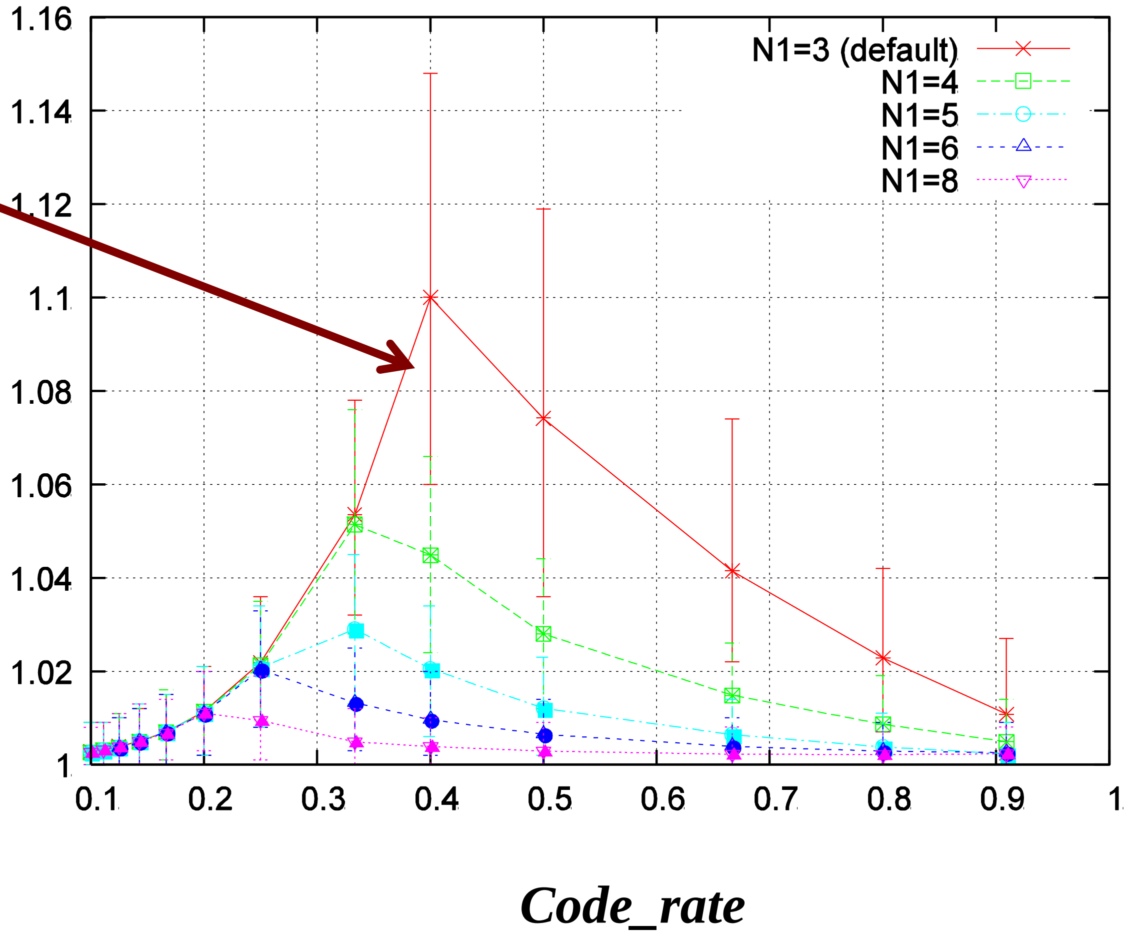
- Increasing N1 …
  - **… improves the erasure correction capabilities**

Perfs with the default

Value, N1=3

Average inefficiency ratio

(the lower, the better)



LDPC-Staircase,

Object_size =1000 symbols

Improved erasure correction capabilities

- LDPC-staircase results (N1=5, k=1,000)

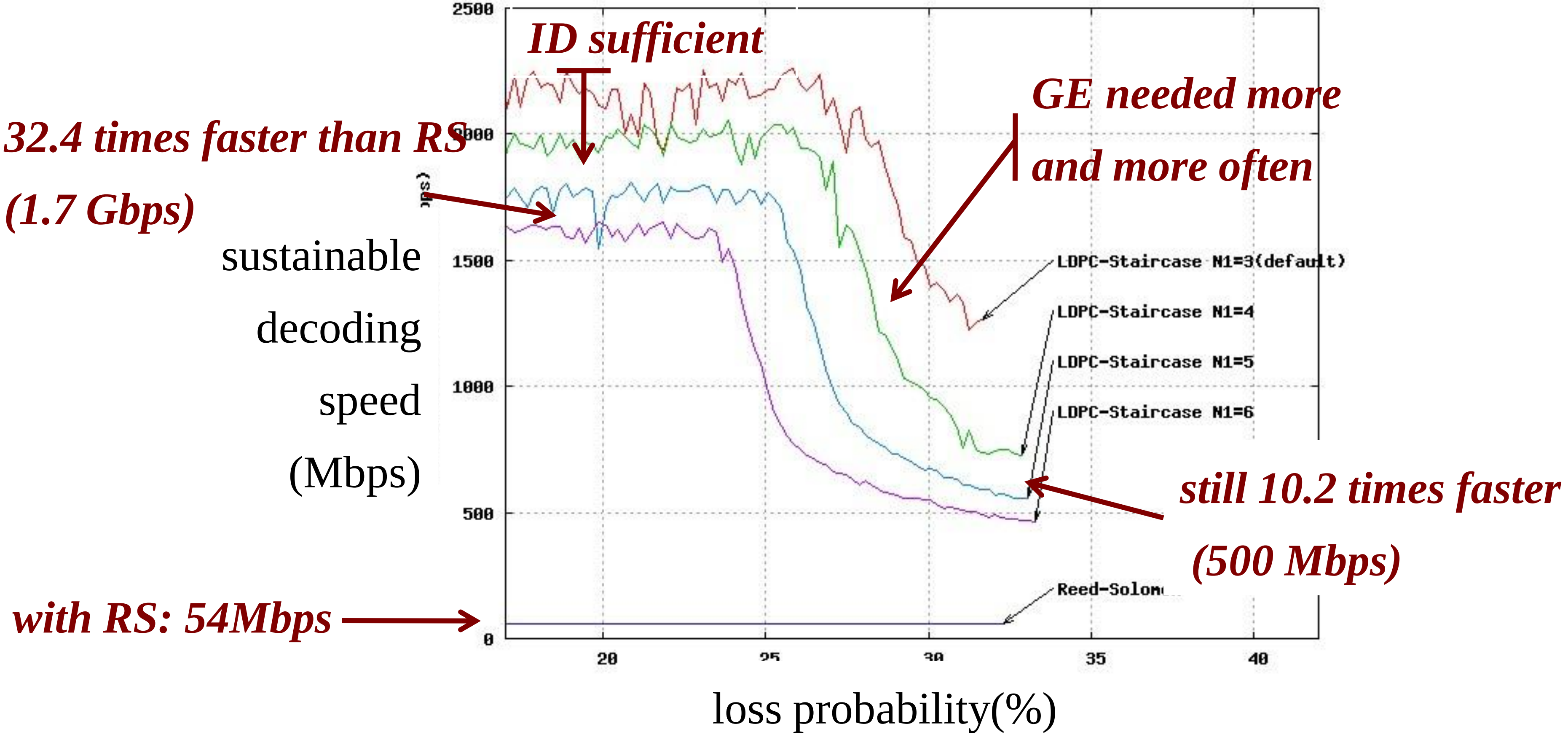
code rate	average overhead	overhead for a failure proba ≤ 10^{-4}
2/3 (=0.66)	0.63%	2.21%
2/5 (=0.4) (worst case!)	2.04%	4.41%

- **then erasure correction capabilities further improve as the code rate decreases**
  - means that **small-rate codes are feasible**…

- **erasure correction capabilities remain excellent with smaller objects (<1000 symbols)**

Decoding Speed

- LDPC-staircase, code rate 2/3, k=1,000
  - the higher the N1, the lower the decoding speed
  - yet with N1=5, between 32 to 10 times faster than Reed Solomon codes over GF(2^8)



ID sufficient

GE needed more and more often

32.4 times faster than RS (1.7 Gbps)

still 10.2 times faster (500 Mbps)

with RS: 54Mbps

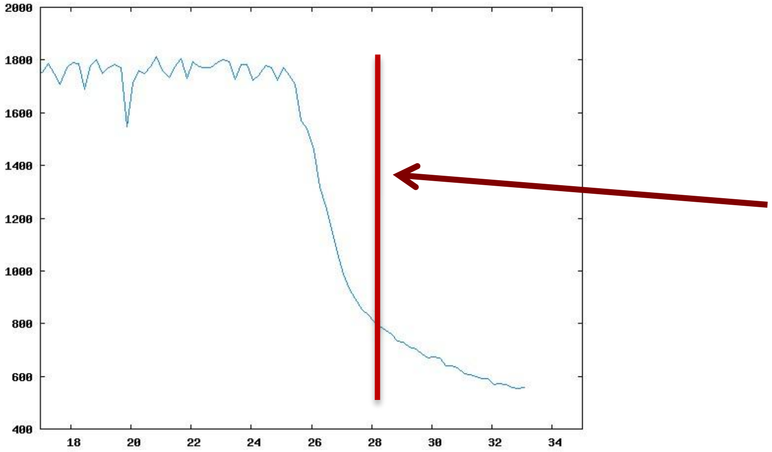
Decoding Speed

- These results were obtained in June …
- … progress has been done and …
- … improvement of the GE are on the way
  - Reduce the decoding complexity
  - Increase the decoding speed (x5 expected)
  - Make GE feasible for larger object

Flexibility of hybrid decoding

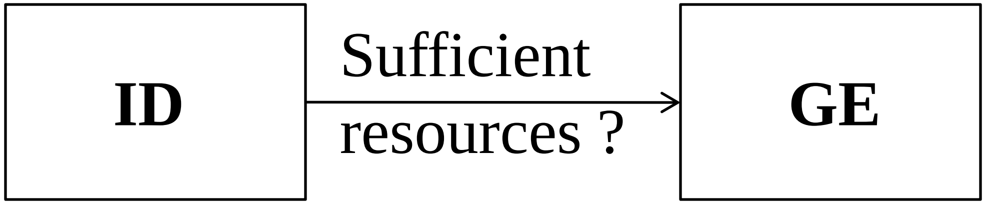
- A highly flexible decoding scheme
  - use of the GE decoding as a function of :
    - **available resources (computation power, battery …)**
    - **complexity of the GE decoding (size of the system)**

sustainable decoding speed (Mbps)



loss probability(%)

Threshold
(adapted as a function of the available resources)



ID

Sufficient resources ?

GE

To conclude

- Excellent results of LDPC-Staircase codes:
  - with blocks that are between a few 100s and a few 1,000s symbols long
    - **close to ideal codes**
    - **while remaining rather fast and highly flexible**

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- Do we really need anything else ?
  - For fixed rate codes, probably not…
  - Erasure recover can be marginally improved, it won’t really make a difference!

Questions ?